WEL02 DIFFRACTION OF LIGHT

SPH4U

CH 10 (KEY IDEAS)

- describe polarized light in terms of its properties and behaviour and how it is applied in everyday applications
- explain single-slit diffraction and diffraction grating interference patterns, both qualitatively and quantitatively
- explain the operation of the spectroscope and the interferometer in terms of the wave properties of light
- describe how the wave properties of light are important in resolution of optical instruments and how these properties are applied in various applications of thin-film interference, for example: Newton’s rings, colours in thin films, coated surfaces, CDs, and DVDs
- explain the basic concepts holography
- describe electromagnetic waves in terms of their properties and where they belong in the electromagnetic spectrum

EQUATIONS

- Minima for Single Slit

$$\sin\theta_n = \frac{n\lambda}{w}, n = 1, 2, 3, \ldots$$

- Maxima for Single Slit

$$\sin\theta_m = \frac{\left(m + \frac{1}{2}\right)\lambda}{w}, m = 0, 1, 2, 3, \ldots$$

- Separation of Adjacent Maxima or Minima for Single Slit

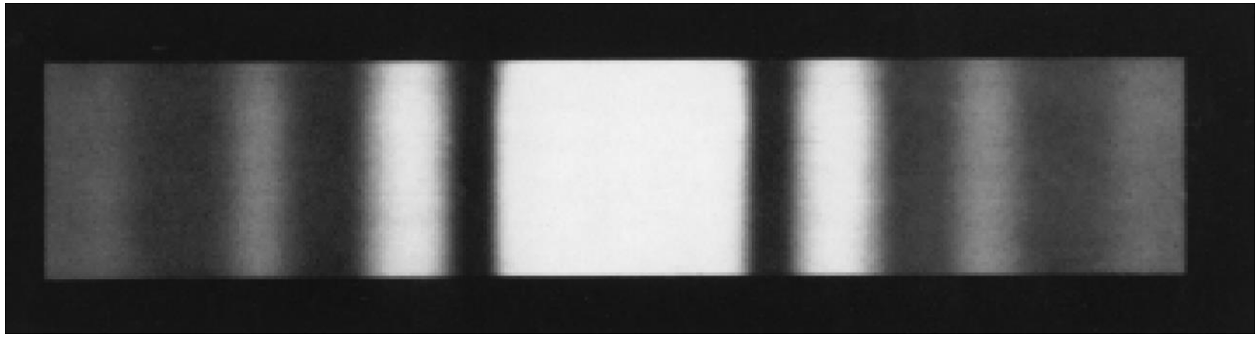
$$\Delta y = \frac{\lambda L}{w}$$

- Maxima for Diffraction Gratings

$$\sin\theta_m = \frac{m\lambda}{d}, m = 0, 1, 2, 3, \ldots$$

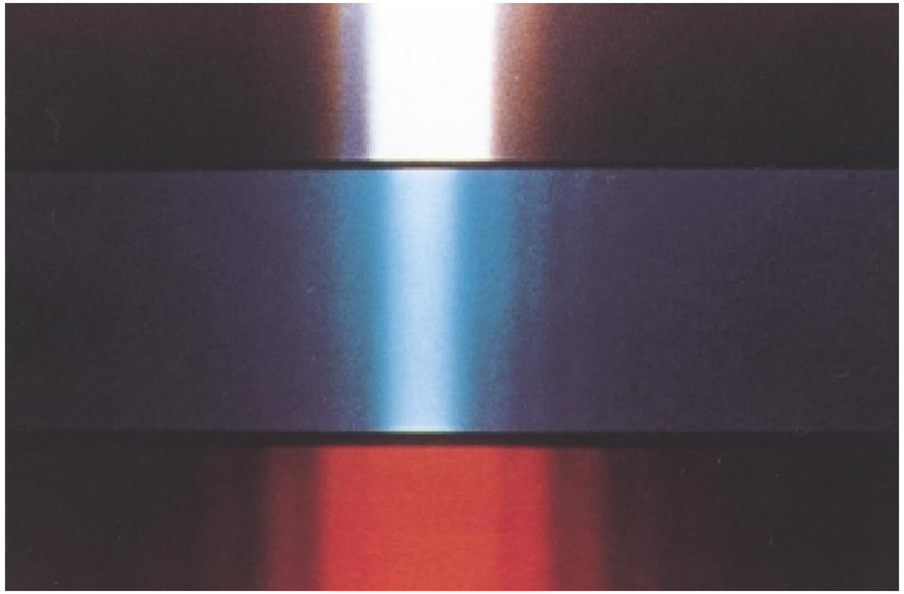
DIFFRACTION OF LIGHT THROUGH A SINGLE SLIT – CONT.

- **Central Maximum:** the bright central region in the interference pattern of light and dark lines produced in diffraction
- **Secondary Maxima:** the progressively less intense bright areas, outside the central region, in the interference pattern
- Note: the spacing and intensity of the maxima are not uniform, with the central maximum being considerably brighter and wider



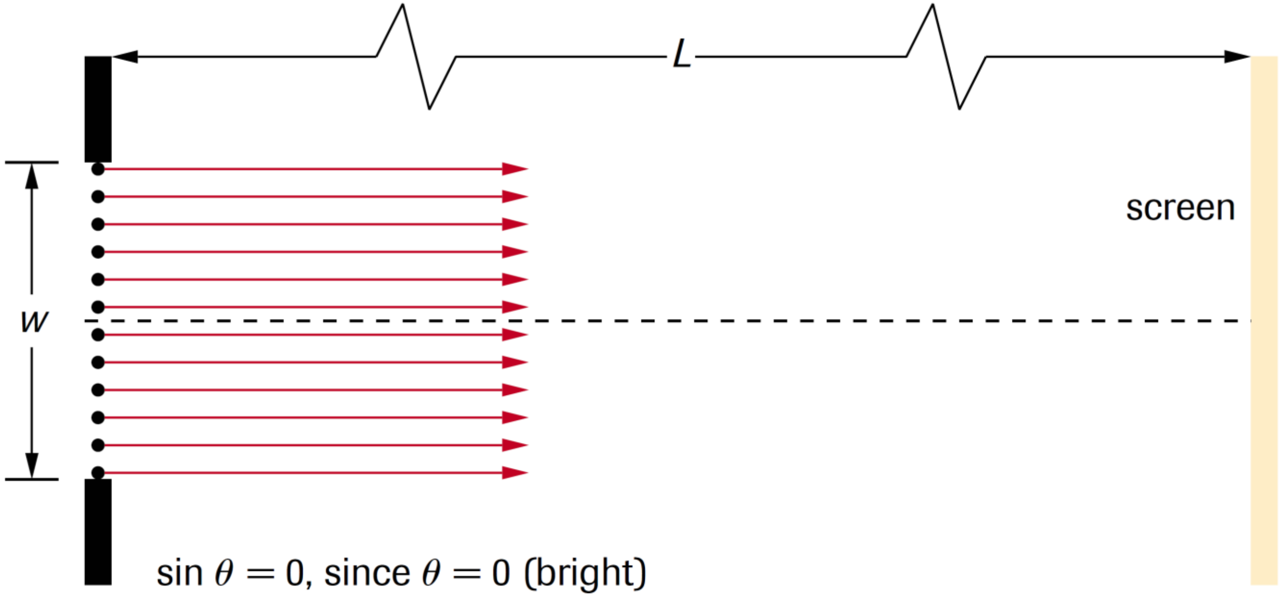
DIFFRACTION OF LIGHT THROUGH A SINGLE SLIT – CONT.

- Different wavelengths display different fringe patterns
- This causes the various colours of white light to disperse
  - Red Light: longer wavelength, wider central maximum
  - Blue Light: shorter wavelength, narrower central maxim



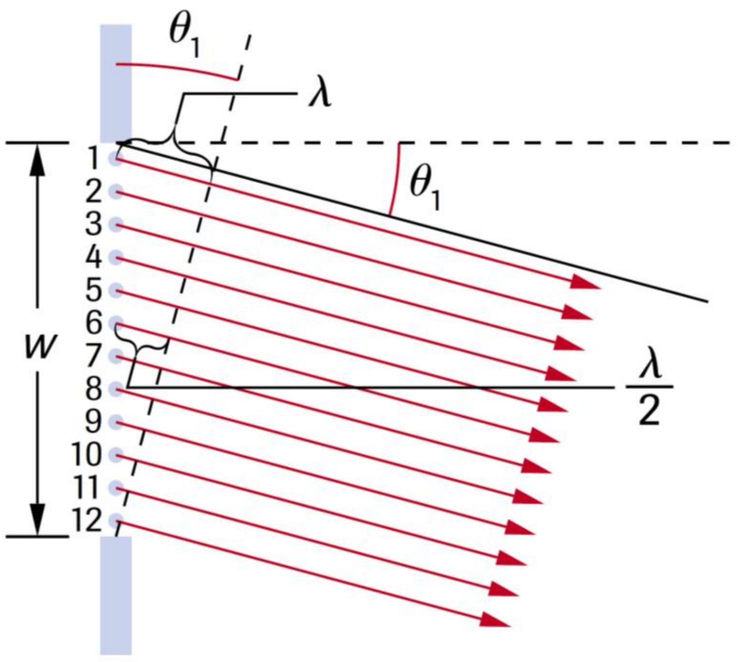
HUYGENS' PRINCIPLE IN SINGLE-SLIT DIFFRACTION

- Recall: Huygens modelled a wavefront as a series of point sources
  - Each of these sources will send light in all directions
- For the central maximum, we are looking all light travelling perpendicular to the slit ($\theta = 0$)
- Constructive interference, as all waves will be in phase and travel the same distance



HUYGENS' PRINCIPLE IN SINGLE-SLIT DIFFRACTION – CONT.

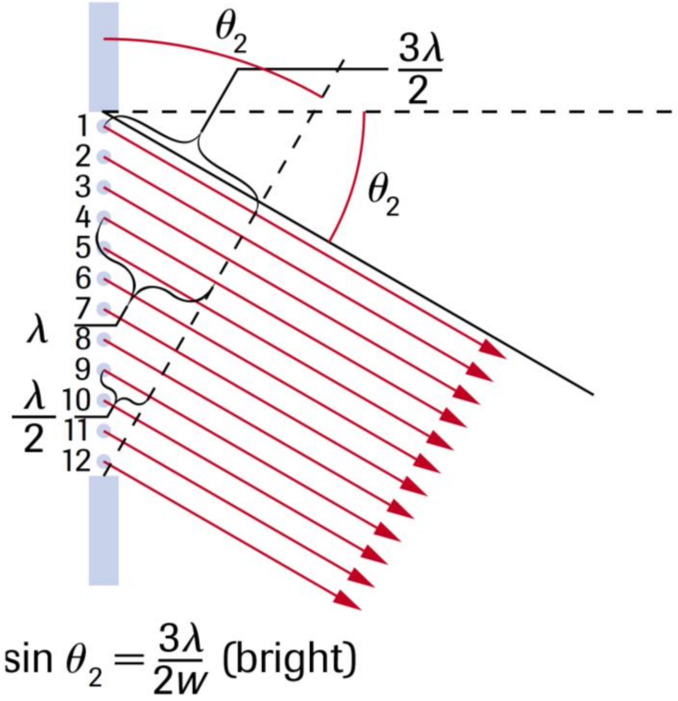
- First Minimum: this is the first area of destructive interference
  - Recall: destructive interference occurs when intersecting wavelengths are in anti-phase
- Similar to our interference patterns with two slits, we approximate the waves as parallel close to the slit and converging at the screen
  - the slit is very small and the screen is very far
- A key difference is the path difference
  - Destructive interference occurs when rays from the top and bottom of the slit have a path difference of λ, since they interfere with the central rays with a path difference of $\frac{\lambda}{2}$



$\sin\theta_1 = \frac{\lambda}{w}$ (dark)

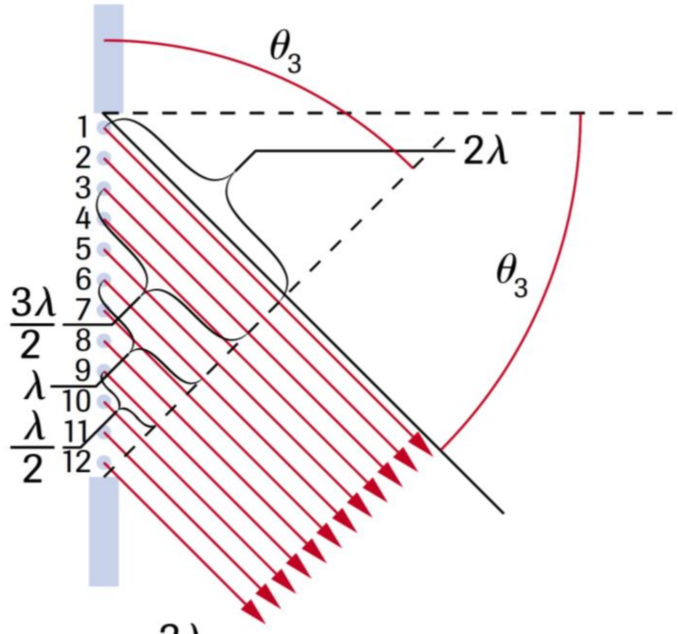
HUYGENS' PRINCIPLE IN SINGLE-SLIT DIFFRACTION – CONT.

- When the path difference between the top and bottom of the slit is $\frac{3}{2}\lambda$, we see some rays interfere destructively
  - This reduces the intensity of the maximum
- All other rays will interfere without fully cancelling, allowing the first secondary maximum to form



HUYGENS' PRINCIPLE IN SINGLE-SLIT DIFFRACTION – CONT.

- When the angle the light passes through the slit is large enough, the path difference between the top and bottom is 2λ
- For every ray in the upper half of the slit, there is a corresponding ray in the lower half that will cancel it out
- We see total destructive interference when the light reaches the screen, forming the second minimum



$\sin\theta_3 = \frac{2\lambda}{w}$ (dark)

DIFFRACTION MINIMA AND MAXIMA

- Destructive interference resulting in zero intensity (minima or dark fringes) occurs when

$$\sin\theta_n = \frac{n\lambda}{w}, n = 1, 2, 3, \ldots$$

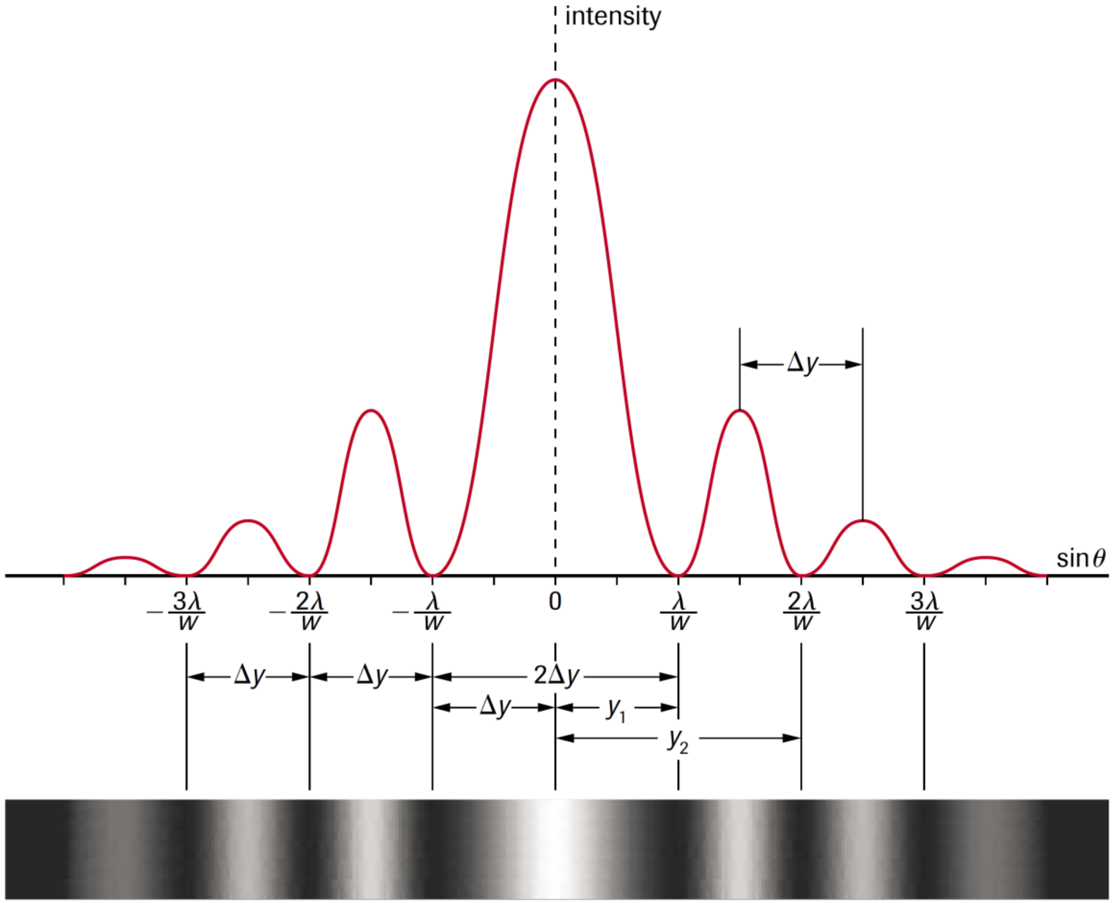
- Constructive interference resulting in maximum intensity (maxima or bright fringes) occurs when

$$\sin\theta_m = \frac{\left(m + \frac{1}{2}\right)\lambda}{w}, m = 1, 2, 3, \ldots$$

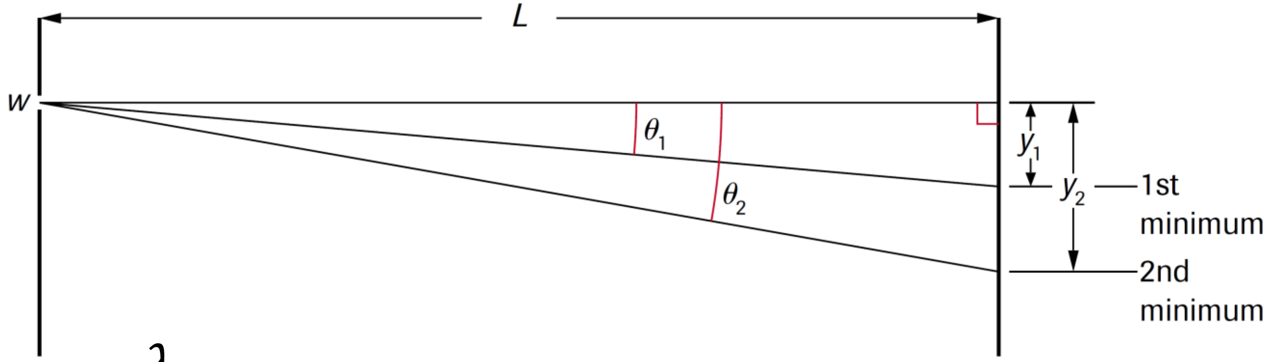
- Note: the central maximum occurs with zero path difference; there is no “zero order” secondary maximum

DIFFRACTION PATTERN FOR SINGLE SLIT

- With close approximation, we can say that the separation of each minima is equal, denoted by Δy
- This is the same as the distance between secondary maxima
- The central maxima is twice the width, as goes between a minimum to the centre then to the next minimum $(2\Delta y)$



FINDING λ



- The width of the central maximum is dependent on the positions of the first minima
- From before, we know

$$\sin \theta_1 = \frac{\lambda}{w}$$

- Similar to the math used for interference patterns, this becomes

$$\frac{y_1}{L} = \frac{\lambda}{w}$$

- Since the distance y_1 is the same as the distance between two minima Δy, we can write the wavelength as

$$\boldsymbol{\lambda = \frac{w\Delta y}{L}}$$

PROBLEM 1

Light from a laser pointer, with a wavelength of 6.70×10^2 nm, passes through a slit with a width of 12 μm. A screen is placed 30 cm away.

(a) How wide is the central maximum (i) in degrees and (ii) in centimetres?

(b) What is the separation of adjacent minima (excluding the pair on either side of the central maximum)?

PROBLEM 1 – SOLUTIONS

(a) $\lambda = 6.70 \times 10^2 \text{ nm} = 6.70 \times 10^{-7} \text{ m}$ $\quad n = 1$

$w = 12\ \mu\text{m} = 1.2 \times 10^{-5} \text{ m}$ $\quad \theta = ?$

$L = 0.30 \text{ cm}$

(i) On either side of the central line, the width of the central maximum is defined by the first-order dark fringes. Thus,

$$\sin\theta_1 = \frac{\lambda}{w}$$

$$= \frac{6.70 \times 10^{-7} \text{ m}}{1.2 \times 10^{-5} \text{ m}}$$

$$\sin\theta_1 = 5.58 \times 10^{-2}$$

$$\theta_1 = 3.2^\circ$$

The angular width of the central maximum is $2 \times 3.2^\circ = 6.4^\circ$.

PROBLEM 1 – SOLUTIONS CONT.

(ii)

$$\sin \theta_1 = \frac{y_1}{L}$$

$$y_1 = L \sin \theta_1$$

$$= (0.30 \text{ m}) \sin 3.2°$$

$$y_1 = 1.67 \times 10^{-2} \text{ m, or } 1.7 \text{ cm}$$

The width of the central maximum is $2y_1$, or 2×1.67 cm $= 3.3$ cm.

(b)

$$\lambda = \frac{w\Delta y}{L}$$

$$\Delta y = \frac{L\lambda}{w}$$

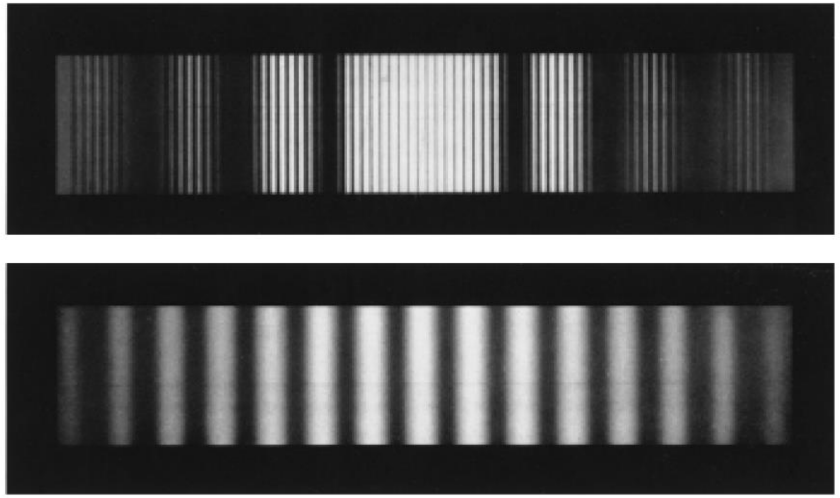
$$= \frac{(0.30 \text{ m})(6.70 \times 10^{-7} \text{ m})}{1.2 \times 10^{-5} \text{ m}}$$

$$\Delta y = 1.7 \times 10^{-2} \text{ m, or } 1.7 \text{ cm}$$

The separation of adjacent minima is 1.7 cm. The central maximum is exactly twice the width of the separation of other adjacent dark fringes.

YOUNG'S EXPERIMENT WITH THE SINGLE SLIT

- In reality, Young's Double-Slit interference is the resultant interference of two single-slit interference patterns
- The secondary maxima are usually too fain to see, but can appear when the slit is sufficiently wide

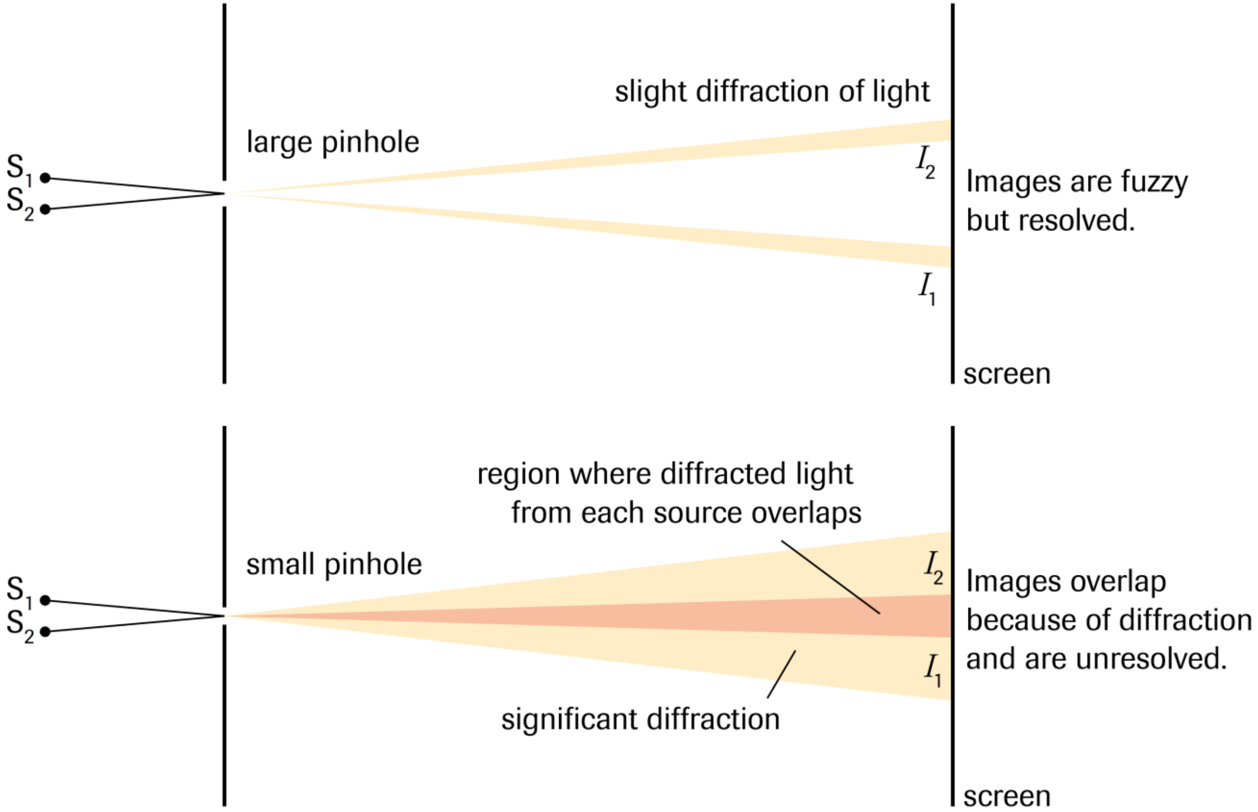


EXTRA RESOURCES

- The following Khan Academy videos can help you understand Single-Slit Interference
  - https://www.youtube.com/watch?v=7CmbItRjM-Y
  - https://www.youtube.com/watch?v=T-kgoxhFSmU

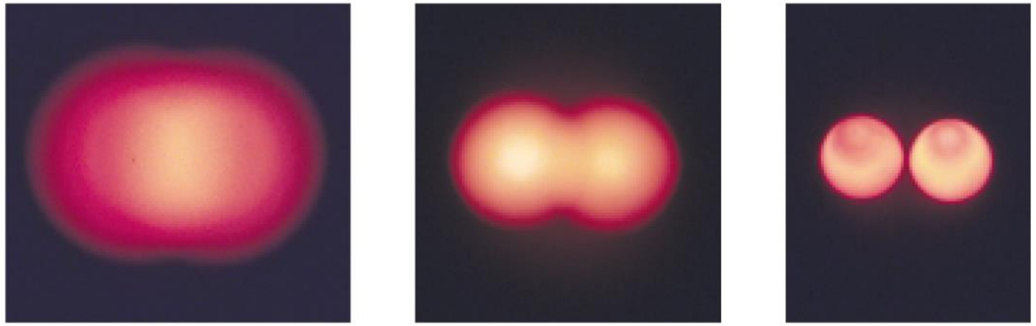
RESOLUTION

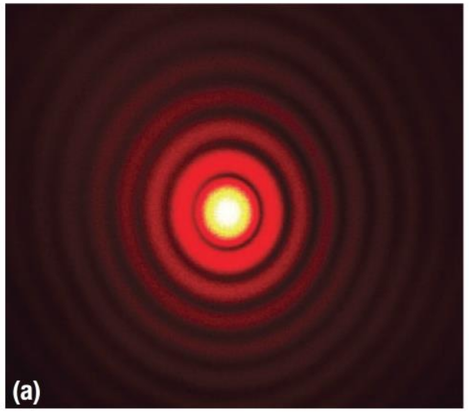
- **Resolution:** the ability of an instrument to separate two images that are close together
- When light from two sources passes through a small hole, the light diffracts and overlaps, causing the images to be unresolvable

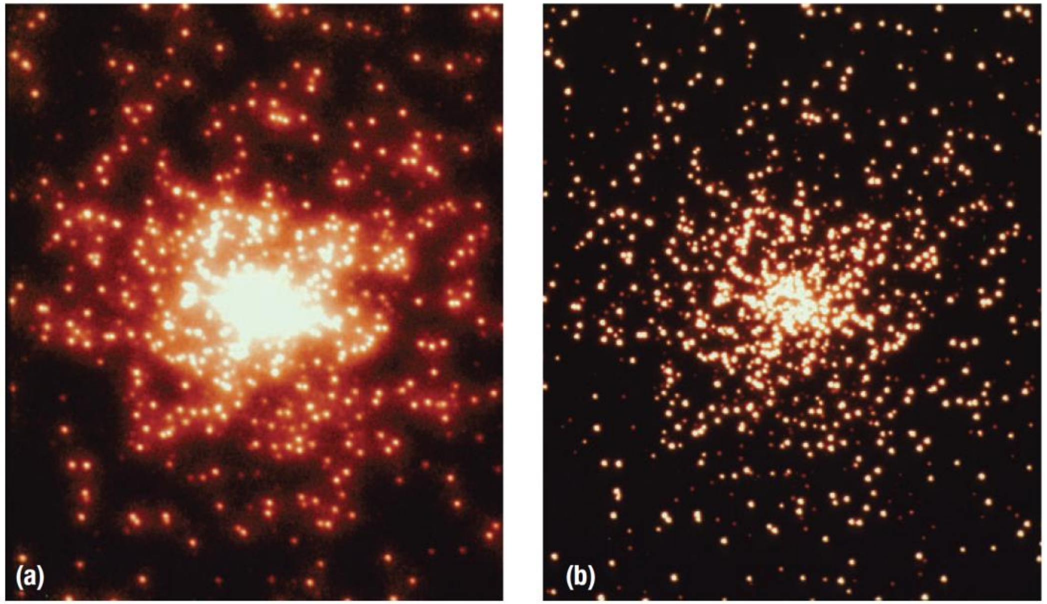


RESOLUTION – CONT.

- When objects are unresolved, they show a pattern similar to single-slit diffraction
- This pattern is called an Airy disk (named after British Astronomer Sir George Airy)

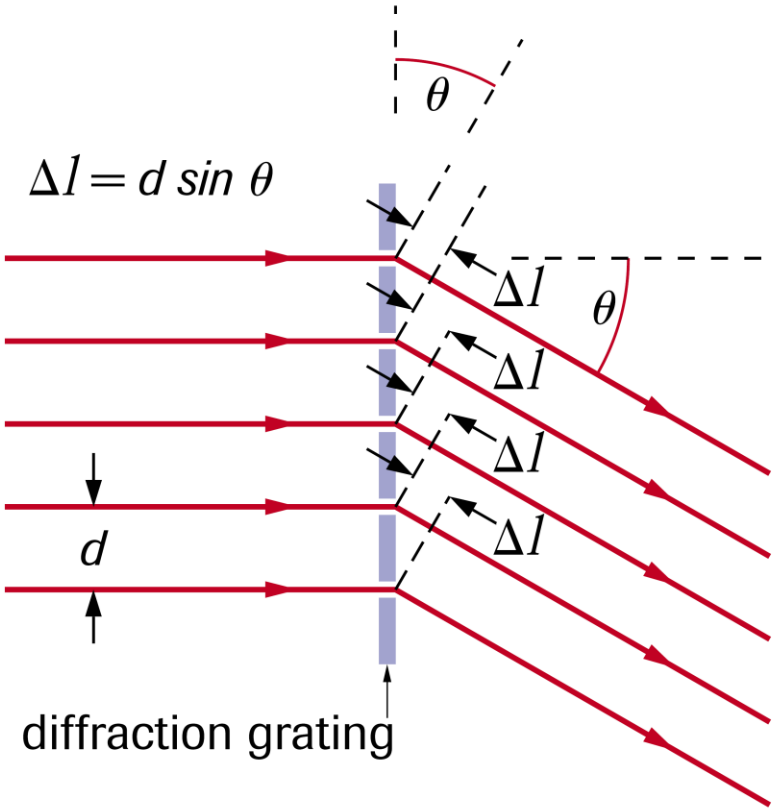






DIFFRACTION GRATINGS

- **Diffraction Grating:** device whose surface is ruled with close, equally spaced, parallel lines for the purpose of resolving light into spectra; transmission gratings are transparent; reflection gratings are mirrored
- Note: two slits technically makes a grating, but most gratings have many more slits than this



DIFFRACTION GRATINGS – CONT.

- We use the same formula for bright fringes (maxima) as for the double slit

$$\sin \theta_m = \frac{m\lambda}{d}, m = 0, 1, 2, 3, \ldots$$

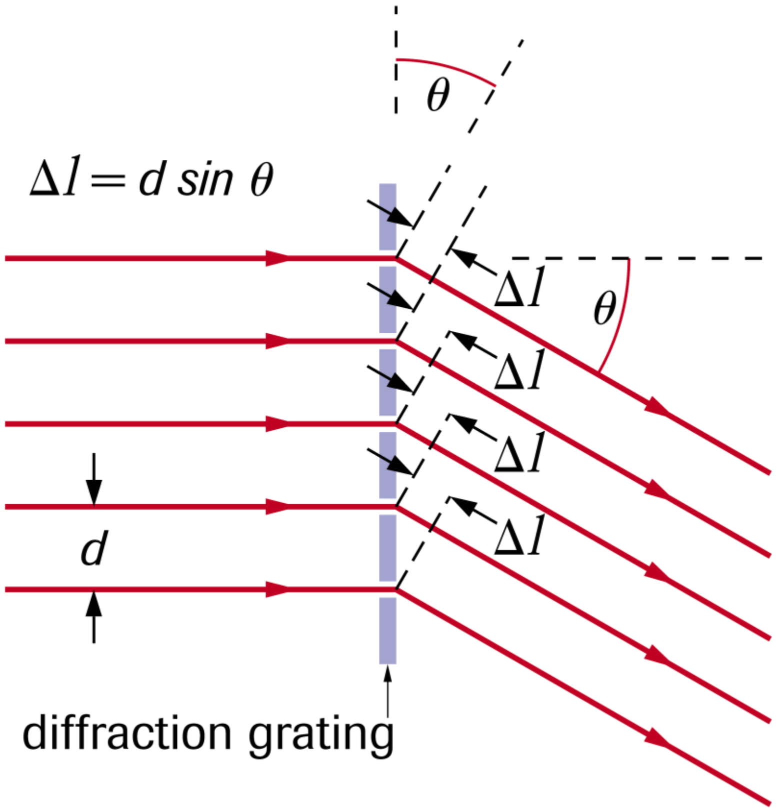
- The path difference Δl can be found with

$$\Delta l = d \sin \theta$$

- The slit separation d can be found with

$$d = \frac{1}{N}$$

where N is the number of lines per unit (usually cm)



PROBLEM 2

At what angle will 638-nm light produce a second-order maximum when passing through a grating of 900 lines/cm?

PROBLEM 2 – SOLUTIONS

$\lambda = 638 \text{ nm} = 6.38 \times 10^{-7} \text{ m}$ $\quad d = \dfrac{1}{900 \text{ lines/cm}} = 1.11 \times 10^{-3} \text{ cm} = 1.11 \times 10^{-5} \text{ m}$

$m = 2$ $\quad \theta = ?$

$$\sin \theta_m = \frac{m\lambda}{d} \quad \text{(for bright maxima)}$$

$$\sin \theta_2 = \frac{2(6.38 \times 10^{-7} \text{ m})}{1.11 \times 10^{-5} \text{ m}}$$

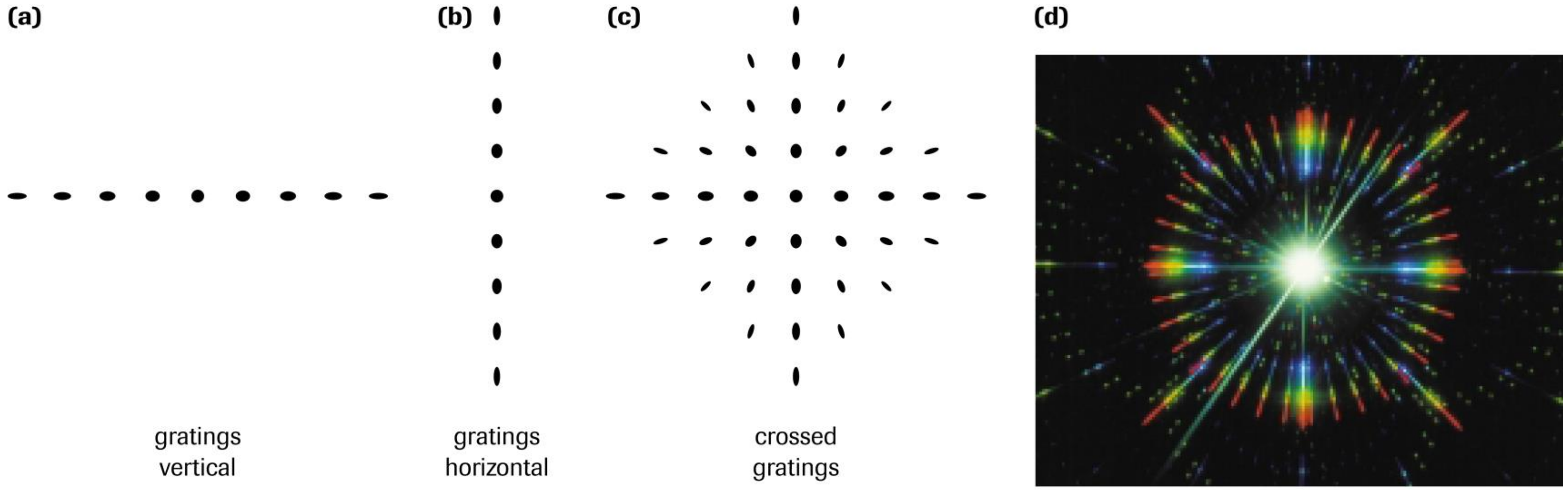
$$\sin \theta_2 = 1.15 \times 10^{-1}$$

$$\theta_2 = 6.60°$$

The angle to the second maximum is 6.60°.

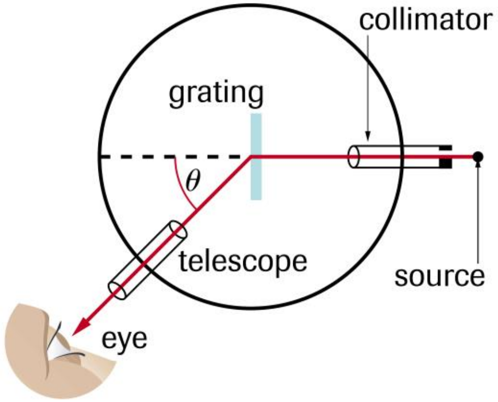
CROSSED GRATINGS

- Gratings can be oriented vertically or horizontally, with the diffraction pattern oriented perpendicular to the grating
- By combining vertical and horizontal gratings, we see a crossed grating pattern, as shown below



CROSSED GRATINGS AND THE SPECTROSCOPE

- **Spectroscope:** an instrument that uses a diffraction grating to visually observe spectra
- Light passes through a grating and the diffraction pattern is analyzed by a telescope
- This is useful in determining which wavelengths are emitted by different sources



SUMMARY – DIFFRACTION OF LIGHT THROUGH A SINGLE SLIT

- Light passing through a single slit creates a diffraction pattern. The pattern consists of a bright central region with dark regions of destructive interference, alternating with progressively less intense areas of bright constructive interference.
- The smaller the slit width, the larger the distance between maxima and minima.
- The longer the wavelength, the greater the distance between maxima.
- Minima, or dark fringes, occur at $\sin\theta_n = \frac{n\lambda}{w}$ $(n = 1, 2, \ldots)$
- Maxima, or bright fringes, occur at the centre of the pattern and also at $\sin\theta_m = \frac{\left(m+\frac{1}{2}\right)\lambda}{w}$ $(m = 1, 2, \ldots)$
- The separation Δy of adjacent maxima or minima is given by the relationship $\Delta y = \frac{\lambda L}{w}$, and the central maximum width is $2\Delta y$.
- Young's double-slit interference pattern results from the interference of two single-slit diffraction patterns.
- Resolution, or the ability of an instrument to separate two closely spaced images, is limited by the diffraction of light.

SUMMARY – DIFFRACTION GRATINGS

- The surface of a diffraction grating consists of a large number of closely spaced, parallel slits.
- Diffraction gratings deliver brighter interference patterns than typical double-slit setups, with maxima that are narrower and more widely separated.
- Diffraction gratings are governed by the relationship $\sin\theta_m = \frac{m\lambda}{d}$, where d is the distance between adjacent gratings, and m is the order of the maxima.

PRACTICE

Readings

- Section 10.2 (pg 499)
- Section 10.3 (pg 508)

Questions

- pg 507 #1,3,5,7,9
- pg 511 #1-5